Direct and dual elasticities of substitution under non-homogenous technology and nonparametric distribution

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Abstract

Purpose – This paper revisits the derivation and properties of the Allen-Uzawa and Morishima elasticities. Using a Swiss dataset, this paper empirically estimates various elasticities both in a dual and primal framework using a production theory open economy model and tests for linear homogenous technology. In addition to reporting elasticity at the mean, the standard practice in the literature, this paper also calculates nonparametric distribution of various elasticities. The paper aims to discuss these issues.

Design/methodology/approach – To assess the effect of price change on input, the paper estimates a translog cost function and to assess the effect of quantity change on price, the paper estimates the translog distance function using the data on Swiss economy. The paper estimates Allen-Uzawa and Morishima elasticity both under homogenous and non-homogenous technology using the Swiss dataset of one aggregate gross output and four inputs (resident labor, non-resident labor, imports, and capital) over 1950-1986. Elasticities are reported and compared at the mean as well as explored by looking at the range and nonparametric distribution.

Findings – This paper shows that constant returns to scale are easily rejected in this dataset and that the elasticities, both qualitatively and quantitatively, are very different under homogenous and non-homogenous technology. These elasticities can switch from complements to substitutes or vice versa when one moves away from the mean of the sample. The equality of the nonparametric elasticity distributions under homogenous vs non-homogenous technology is rejected in all cases except one.

Originality/value – This paper gives a clear derivation and interpretation of different elasticities as well as demonstrates using a dataset how to systematically go about empirically estimating these elasticities in a dual and primal framework. It shows that linear homogenous technology can be easily rejected and the elasticities, both quantitatively and qualitatively, are very different under homogenous and non-homogenous technology. This paper is also very valuable because it shows that the standard practice of reporting elasticity at the mean might not be adequate and there is a possibility that these elasticities can switch from complements to substitutes or vice versa when one moves away from the mean of the sample.

Keywords Allen-Uzawa and Morishima, Direct and dual, Elasticities of substitution and complementarity, Non-homogeneity, Nonparametric distribution

Paper type Research paper

JEL classification – D2, J0

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1. Introductory remarks

The elasticity of substitution was introduced by Hicks (1932) and since then thousands of elasticities have been estimated in different areas of economics. First, this paper revisits the derivation and properties of different elasticities of substitution. We estimate these elasticities both under homogenous and non-homogenous technology using a Swiss dataset on resident and non-resident worker. In the primal quantity space we use the translog distance function and in the dual price space we use the translog cost function. Second, this paper shows how the assumption of linear homogenous technology is often rejected and the usual practice of reporting elasticity at the mean under two different specification of technology often gives very different results. Last, but not the least this paper illustrates the use of the nonparametric frequency distribution for elasticities and compares the probability density functions for various elasticity's under linear homogenous and non-homogenous technology. All the papers in this literature report the elasticities at the mean (Berndt and Wood, 1975; Davis and Gauger, 1996; Grossman, 1982; Kohli, 1999; Kim, 2000 to name a few). However, elasticities reported at the mean may not hold at various levels of inputs and for different time periods and the use of nonparametric frequency distribution might give more insight into the elasticities. Since there are many generalizations of elasticity measures and still some confusion on which elasticity needs to be used we begin with an introduction generalizing Hicks two input elasticity of substitution.

1.1 Generalization of Hicksian elasticity of substitution

Hicks (1932) introduced elasticity of substitution as a tool for analyzing capital and labor income shares in a growing economy with a constant-returns-to-scale technology and neutral technological change. Defined as the logarithmic derivative of the capital/labor ratio with respect to the technical rate of substitution of labor for capital, the elasticity is higher the "easier" is the substitution of one input for the other (the lesser is the degree of "curvature" of the isoquant). Under the assumption of cost-minimizing, price-taking behavior, it is the logarithmic derivative of the capital/labor ratio with respect to the factor-price ratio (the ratio of the wage rate to the rental rate on capital), and it yields immediate (differential) qualitative and quantitative comparative-static information about the effect on relative income shares of changes in factor price ratios.

Under the Hicks two-factor elasticity of substitution, the inverse of the elasticity – the logarithmic derivative of the technical rate of substitution of labor for capital (the factor shadow-price ratio) with respect to the capital/labor ratio – contains the same information, but larger values indicate "less easy" substitution of the two factors for one another (greater "curvature" of the isoquant). Under the assumption of competitive factor markets, it yields immediate comparative-static information about the effect on (absolute and relative) income shares of changes in factor quantity ratios.

Thus, whether one is interested in the effects of changes in factor-price ratios on factor-quantity ratios or the effects of changes in quantity ratios on price ratios (and in each case the effects on relative factor shares), the Hicksian two-factor elasticity of substitution provides complete (differential) qualitative and quantitative comparative-static information.

Matters get more complicated when one's analysis of substitutability and the comparative statics of relative income shares entails more than two factors of production. Prominent examples in the literature include the analyses of:

- the effect of energy-cost explosions using KLEM (capital, labor, energy, materials) data (Berndt and Wood, 1975; Davis and Gauger, 1996; Thompson and Taylor, 1995);
- the effect of increases in human capital (or in educational attainment) on the relative wages of skilled and unskilled labor, with capital as a third important input (Griliches, 1969; Johnson, 1970; Kugler *et al.*, 1989; Welch, 1970);
- the substitutability of (multiple) monetary assets (Barnett *et al.*, 1992; Davis and Gauger, 1996; Ewis and Fischer, 1984);
- the elasticity of substitution between capital and labor in a convex endogenous growth model where the elasticity of substitution may interact with the level of development (Karagiannis *et al.*, 2005; Palivos and Karagiannis, 2010); and
- the effect of immigration on the relative wages of domestic and immigrant labor (Grossman, 1982; Borjas, 1994; Borjas *et al.*, 1992, 1996) or the effect of increase in the number of guest workers on resident and non-resident labor (Kohli, 1999).

In the literature there exist more than one generalization of the Hicksian two-variable elasticity of substitution. Allen and Hicks (1934) and Allen (1938) suggested two generalizations. One, further analyzed by McFadden (1963), eventually lost favor because it does not allow for optimal adjustment of all inputs to changes in factor prices. The other, further analyzed by Uzawa (1962), became the dominant concept; perhaps tens of thousands of "Allen-Uzawa elasticities" have been estimated. Later, Morishima (1967) and Blackorby and Russell (1975, 1981, 1989) proposed an alternative to the Allen-Uzawa elasticity; the latter argued that this "Morishima elasticity" has attractive properties not possessed by the Allen-Uzawa elasticity. Recently, the Morishima elasticity has been gaining favor (Davis and Gauger, 1996; de la Grandville, 1997; Klump and de la Grandville, 2000; Ewis and Fischer, 1984; Flaussig, 1997; Thompson and Taylor, 1995; Stern, 2010, 2011; Kim, 1992)[1].

When one advances to more than two inputs, the measurement of the effect of changes in price ratios on quantity ratios and the effect of changes in quantity ratios on price ratios are not simple inverses of one another. The Allen-Uzawa elasticity or what Stern (2011) calls net p elasticity is formulated in terms of effects of price changes on input demands, but many issues revolve around the effect of quantity changes on price ratios (e.g. the effect of immigration or of increases in the number of guest workers on relative wages or the effect of increases in the number of skilled workers relative to unskilled workers on relative wages or return to education). Hicks (1970) suggested a dual to the Allen-Uzawa elasticity, formulated in terms of a scalar-valued, linearly homogeneous production function. Blackorby and Russell (1981) formulated elasticity of complementarity to both the Allen-Uzawa and the Morishima elasticity of substitution using the distance function, which is symmetrically dual to the cost function employed by Uzawa to reformulate the Allen elasticity. Kim (2000) calls these Antonelli elasticity of complementarity and Stern (2010) net $q-$ complements. For further details on the classification of different elasticities, see Bertoletti (2005) and Stern (2011).

In the duality literature it is well known that the primal space is the quantity space and the dual is the price space, following Diewert (1971) and Stern (2011) we call the elasticity calculated with distance function as the direct elasticities of substitution giving the effect of a change in quantity on price. We call the elasticity of substitution using the cost function as the dual elasticity of substitution giving the effect of a price change on the quantity. The Allen-Uzawa elasticity or what we call dual elasticity are well defined for non-homogeneous production technologies with multiple outputs as well as multiple inputs. The elasticities of complementarity or what we call the direct elasticity using the distance function are also well defined for multiple-output, non-homogeneous production technologies. Non-homogeneity is an especially important property when more than two inputs are employed, because it is typically easy to reject homogeneity for production technologies with more than two inputs.

The next section gives the representation of the technology and briefly summarizes the expression and interpretation for the elasticity in the direct and dual framework both for the Allen-Uzawa and the Morishima elasticitites[2]. Given that there are many terminologies in the elasticity literature this section is important for clarifying and reconciling different concepts and terminologies in the literature. Section 3 describes a method of estimating these elasticities using, alternatively, a translog cost function and a translog distance function and applies these concepts to the Swiss data on resident and non-resident (guest) labor (and other inputs). Section 4 discusses the representation of these elasticities using nonparametric frequency distributions and discusses the variability of these elasticities between inputs and the differences from the mean. Section 5 concludes.

2. Elasticities of substitution and complementarities

2.1 Representations of technologies

Input and output quantity vectors are denoted $x \in \mathbf{R}_+^n$ and $y \in \mathbf{R}_+^m$, respectively. The technology set is the set of all feasible ⟨input, output⟩ combinations:

$$T := \left\{ \langle x, y \rangle \in \mathbf{R}_+^{n+m} | x \ \text{can produce} \ y \right\}.$$

While the nomenclature suggests that feasibility is a purely technological notion, a more expansive interpretation is possible: feasibility could incorporate notions of institutional and political constraints, especially when we consider entire economies as the basic production unit. An input requirement set for a fixed output vector y is:

$$L(y) := \left\{ x \in \mathbf{R}_+^n | \langle x, y \rangle \in T \right\}. \tag{2.1}$$

We assume throughout that $L(y)$ is closed, strictly convex, and twice differentiable[3] for all $y \in \mathbf{R}_+^m$ and satisfies strong input disposability[4], output monotonicity[5], and "no free lunch"[6].

The (input) distance (gauge) function, a mapping from[7]:

$$Q := \left\{ \langle x, y \rangle \in \mathbf{R}_+^{n+m} | y \neq 0^{(m)} \wedge x \neq 0^{(n)} \wedge L(y) \neq \emptyset \right\}$$

into the positive real line (where $0^{(n)}$ is the null vector of $\mathbf{R}_+^n$), is defined by:

$$D(x, y) := \max\{\lambda | x/\lambda \in L(y)\}. \tag{2.2}$$

Under the above assumptions, D is well defined on this restricted domain and satisfies homogeneity of degree one, positive monotonicity, concavity, and continuity in x and negative monotonicity in y. We assume, in addition, that it is continuously twice differentiable in x. (See, e.g. Färe and Primont (1995) for proofs of these properties and most of the duality results that follow[8]).

The distance function is a representation of the technology, since (under our assumptions):

$$\langle x, y \rangle \in T \Leftrightarrow D(x, y) \geq 1.$$

In the single-output case ($m = 1$), where the technology can be represented by a production function, $f : \mathbf{R}^n_+ \to \mathbf{R}_+$, $D(x, f(x)) = 1$ and the production function is recovered by inverting $D(x, y) = 1$ in y. If (and only if) the technology is homogeneous of degree one (constant returns to scale):

$$D(x, y) = \frac{f(x)}{y}. \tag{2.3}$$

The cost function, $C : \mathbf{R}^n_{++} \times Y \to \mathbf{R}_+$, where:

$$Y = \{ y | \langle x, y \rangle \in Q \ \text{for some} \ x \}, \tag{2.4}$$

is defined by:

$$C(p, y) = \min_x \{ p \cdot x | x \in L(y) \}$$

or, equivalently, by:

$$C(y, p) = \min_x \{ p \cdot x | D(x, y) \geq 1 \}. \tag{2.5}$$

Under our maintained assumptions, D is recovered from C by:

$$D(x, y) = \inf_p \{ p \cdot x | C(p, y) \geq 1 \}, \tag{2.6}$$

and C has the same properties in p as D has in x. This establishes the duality between the distance and the cost function. On the other hand, C is positively monotonic in y. We also assume that C is twice continuously differentiable in p.

By Shephard's lemma (application of the envelope theorem to equation (2.5)), the (vector-valued, constant-output) input demand function, $\delta : \mathbf{R}^n_{++} \times Y \to \mathbf{R}^n_+$, is generated by first-order differentiation of the cost function:

$$\delta(p, y) = \nabla_p C(p, y). \tag{2.7}$$

Of course, δ is homogeneous of degree zero in p. The (normalized) shadow-price vector, $\rho : Q \to \mathbf{R}_+$, is obtained by applying the envelope theorem to equation (2.6):

$$\rho(x, y) = \nabla_x D(x, y). \tag{2.8}$$

As is apparent from the re-writing of equation (2.6) (using homogeneity in p) as:

$$D(x, y) = \inf_{p/c} \left\{ \frac{p}{c} \cdot x | C(p/c, y) \geq 1 \right\} = \inf_{p/c} \left\{ \frac{p}{c} \cdot x | C(p, y) \geq c \right\}, \tag{2.9}$$

where c can be interpreted as (minimal) expenditure (to produce output y), the vector $\rho(x, y)$ in equation (2.8) can be interpreted as shadow prices normalized by minimal cost[9]. In other words, under the assumption of cost-minimizing behavior:

$$\rho(\overset{*}{x}, y) := \rho(\delta(p, y), y) = \frac{p}{C(p, y)}. \tag{2.10}$$

Clearly, ρ is homogeneous of degree zero in x.

2.2 Elasticities of substitution or the dual elasticity

The Allen-Uzawa elasticity of substitution between inputs i and j is given by:

$$\begin{aligned}\sigma_{ij}^{A}(p, y) :&= \frac{C(p, y)C_{ij}(p, y)}{C_i(p, y)C_j(p, y)} \\ &= \frac{\boldsymbol{e}_{ij}(p, y)}{s_j(p, y)},\end{aligned} \tag{2.11}$$

where C is the cost function and the subscripts on C indicate differentiation with respect to the indicated variable(s):

$$\boldsymbol{e}_{ij}(p, y) := \frac{\partial \ln \delta_i(p, y)}{\partial \ln p_j} \tag{2.12}$$

is the (constant-output) elasticity of demand for input i with respect to changes in the price of input j, and:

$$s_j(p, y) = \frac{p_j \delta_j(p, y)}{C(p, y)} \tag{2.13}$$

is the cost share of input j. Details on the representation of technology is given in the Appendix.

The Morishima elasticity of substitution of input i for input j is:

$$\begin{aligned}\sigma_{ij}^{M}(p, y) :&= \frac{\partial \ln (\hat{\delta}i(p^i, y)/\hat{\delta}j(p^i, y))}{\partial \ln (p_j/p_i)} \\ &= p_j\left(\frac{C_{ij}(p, y)}{C_i(p, y)} - \frac{C_{jj}(p, y)}{C_j(p, y)}\right) \\ &= \boldsymbol{e}_{ij}(p, y) - \boldsymbol{e}_{jj}(p, y),\end{aligned} \tag{2.14}$$

where p^i is the $(n-1)$-dimensional vector of price ratios with p_i in the denominator and (with the use of zero-degree homogeneity of δ in p):

$$\hat{\delta}(p^i, y) := \delta(p, y). \tag{2.15}$$

The Morishima elasticity, unlike the Allen-Uzawa elasticity, is non-symmetric, since the value depends on the normalization adopted in equation (2.14) – that is, on the coordinate direction in which the prices are varied to change the price ratio, p_j/p_i (Blackorby and Russell, 1975, 1981, 1989).

If $\sigma_{ij}^{A}(p, y) > 0$ (that is, if increasing the jth price increases the optimal quantity of input i), we say that inputs i and j are Allen-Uzawa dual substitutes; if $\sigma_{ij}^{A}(p, y) < 0$, they are Allen-Uzawa dual complements. Similarly, if $\sigma_{ij}^{M}(p, y) > 0$ (that is, if increasing the jth price increases the optimal quantity of input i relative to the optimal quantity of input j), we say that input j is a dual Morishima substitute for input i; if $\sigma_{ij}^{M}(p,y) < 0$, input j is a dual Morishima complement to input i. As the Morishima elasticity of substitution is non-symmetric, so is the taxonomy of Morishima substitutes and complements.

The conceptual foundations of Allen-Uzawa and Morishima taxonomies of substitutes and complements are, of course, quite different. The Allen-Uzawa taxonomy classifies a pair of inputs as substitutes (complements) if an increase in the price of one causes an increase (decrease) in the quantity demanded of the other, whereas the Morishima concept classifies a pair of inputs as substitutes (complements) if an increase in the price of one causes the quantity of the other to increase (decrease) *relative to the quantity of the input whose price has changed*. For this reason, the Morishima taxonomy leans more toward substitutability (since the theoretically necessary decrease in the denominator of the quantity ratio in equation (2.14) helps the ratio to decline when the price of the input in the denominator increases). Put differently, if two inputs are dual substitutes according to the Allen-Uzawa criterion, theoretically they must be dual substitutes according to the Morishima criterion, but if two inputs are dual complements according to the Allen-Uzawa criterion, they can be either dual complements or dual substitutes according to the Morishima criterion. This relationship can be seen algebraically from equations (2.11) and (2.14). If i and j are dual Allen-Uzawa substitutes, in which case $\boldsymbol{e}_{ij}(p,y) > 0$, then concavity of the cost function (and hence negative semi-definiteness of the corresponding Hessian) implies that $\boldsymbol{e}_{ij}(p,y) - \boldsymbol{e}_{jj}(p,y) > 0$, so that j is a dual Morishima substitute for i. Similar algebra establishes that two inputs can be dual Morishima substitutes when they are dual Allen-Uzawa complements.

Note that, for $i \neq j$:

$$\frac{\partial \ln s_i(p,y)}{\partial \ln p_j} = \boldsymbol{e}_{ij}(p,y) - s_j(p,y) = s_j(p,y)\Big(\sigma_{ij}^{A}(p,y) - 1\Big), \tag{2.16}$$

so that an increase in p_j increases the absolute cost share of input i if and only if:

$$\sigma_{ij}^{A}(p,y) > 1; \tag{2.17}$$

that is, if and only if inputs i and j are sufficiently net p substitutes. Thus, the Allen-Uzawa elasticities provide immediate qualitative comparative-static information about the effect of price changes on absolute shares. To obtain quantitative comparative-static information, one needs to know the share of the jth input as well as the Allen-Uzawa elasticity of substitution.

The Morishima elasticities immediately yield both qualitative and quantitative information about the effect of price changes on relative input shares:

$$\frac{\partial \ln(\hat{s}_i(p^i,y)/\hat{s}_j(p^i,y))}{\partial \ln(p_j/p_i)} = \boldsymbol{e}_{ij}(p,y) - \boldsymbol{e}_{jj}(p,y) - 1 = \sigma_{ij}^{M}(p,y) - 1, \tag{2.18}$$

where (with the use of zero-degree homogeneity of s_i in p) $\hat{s}_i(p^i,y) := s_i(p,y)$ for all i. Thus, an increase in p_j increases the share of input i relative to input j if and only if:

$$\sigma_{ij}^{M}(p,y) > 1; \tag{2.19}$$

that is, if and only if inputs i and j are sufficiently substitutable in the sense of Morishima. Moreover, the degree of departure of the Morishima elasticity from unity provides immediate quantitative information about the effect on the relative factor shares.

2.3 Elasticities of complementarity or the direct elasticity

Using distance function the Morishima elasticity of substitution (proposed by Blackorby and Russell (1975, 1981)) is given by:

$$\overset{*}{\sigma}{}_{ij}^{M}(x,y) := \frac{\partial \ln(\hat{\rho}i(x^{i},y)/\hat{\rho}j(x^{i},y))}{\partial \ln(x_j/x_i)} = x_j\left(\frac{D_{ij}(x,y)}{D_i(x,y)} - \frac{D_{jj}(x,y)}{D_j(x,y)}\right) = \overset{*}{\boldsymbol{e}}_{ij}(x,y) - \overset{*}{\boldsymbol{e}}_{jj}(x,y), \tag{2.20}$$

where D is the distance function and x^{i} is the $(n-1)$-dimensional vector of input quantity ratios with x_i in the denominator and:

$$\overset{*}{\boldsymbol{e}}_{ij}(x,y) = \frac{\partial \ln \rho_i(x,y)}{\partial \ln x_j} \tag{2.21}$$

is the (constant-output) elasticity of the shadow price of input i with respect to changes in the quantity of input j. Analogously, Blackorby and Russell (1981) proposed the following Allen-Uzawa elasticity using the distance function:

$$\overset{*}{\sigma}{}_{ij}^{A}(x,y) = \frac{D(x,y)D_{ij}(x,y)}{D_i(x,y)D_j(x,y)} = \frac{\overset{*}{\boldsymbol{e}}_{ij}(x,y)}{\overset{*}{s}_j(x,y)}, \tag{2.22}$$

where:

$$\overset{*}{s}_j(x,y) = \rho_j(x,y)\cdot x_j \tag{2.23}$$

is the cost share of input j (assuming cost-minimizing behavior).

If $\overset{*}{\sigma}{}_{ij}^{A}(p,y) < 0$ (that is, if increasing the jth quantity decreases the shadow price of input i), we say that inputs i and j are Allen-Uzawa direct substitutes; if $\overset{*}{\sigma}{}_{ij}^{A}(p,y) > 0$, they are Allen-Uzawa direct complements. Similarly, if $\sigma_{ij}^{M}(p,y) < 0$ (that is, if increasing the jth quantity increases the shadow price of input i relative to the shadow price of input j), we say that input j is a direct Morishima substitute for input i; if $\sigma_{ij}^{M}(p,y) > 0$, input j is a direct Morishima complement to input i. Dual elasticities are fundamentally different concepts; indeed, signs in these definitions of direct substitutability and complementarity are reversed from those in the definitions of dual Allen-Uzawa and Morishima substitutes and complements[10].

Interestingly, since the distance function is concave in x, and hence $\overset{*}{\boldsymbol{e}}_{jj}(x,y)$ in equation (2.20) is non-positive, the direct Morishima elasticity leans more toward direct complementarity than does the direct Allen-Uzawa elasticity (in sharp contrast to the primal taxonomy). Similarly, if two inputs are direct Allen-Uzawa complements, they must be direct Morishima complements, whereas two inputs can be direct Allen-Uzawa substitutes but direct Morishima complements.

There exist, of course, dual comparative-static results linking factor cost shares and elasticities of substitution[11]. Consider first the effect of quantity changes on absolute shares (for $i \neq j$):

$$\frac{\partial \ln \overset{*}{s}_i(x,y)}{\partial \ln x_j} = \overset{*}{\varepsilon}_{ij}(x,y) = \overset{*}{\sigma}{}^{A}_{ij}(x,y)\overset{*}{s}(x,y), \tag{2.24}$$

so that an increase in x_j increases the absolute share of input i if and only if:

$$\overset{*}{\boldsymbol{e}}(x,y) > 0 \tag{2.25}$$

or, equivalently:

$$\overset{*}{\sigma}{}^{A}_{ij}(x,y) > 0; \tag{2.26}$$

that is, if and only if inputs i and j are net $-\ q$ Allen-Uzawa complements. Thus, the direct elasticities provide immediate qualitative comparative-static information about the effect of quantity changes on (absolute) shares. To obtain quantitative comparative-static information, one needs to know the share of the jth input as well as the Allen-Uzawa elasticity of substitution. Of course, the (constant-output) elasticity derived from the distance function $\overset{*}{\boldsymbol{e}}_{ij}(x,y)$ yields the same qualitative and quantitative comparative-static information.

Comparative-static information about relative income shares in the face of quantity changes can be extracted from the Morishima elasticity using distance function:

$$\frac{\partial \ln(\tilde{s}_i(x^i,y)/\tilde{s}_j(x^i,y))}{\partial \ln(x_j/x_i)} = \overset{*}{\boldsymbol{e}}_{ij}(x,y) - \overset{*}{\boldsymbol{e}}_{jj}(x,y) - 1 = \overset{*}{\sigma}{}^{M}_{ij}(x,y) - 1, \tag{2.27}$$

where (with the use of zero-degree homogeneity of $\overset{*}{x}$ in x) ${}_i(x^i,y) :=_i (x,y)$. Thus, an increase in x_j increases the share of input i relative to input j if and only if:

$$\overset{*}{\sigma}{}^{M}_{ij}(x,y) > 1; \tag{2.28}$$

that is, if and only if inputs i and j are sufficiently complementary in terms of the direct Morishima elasticity (or elasticity of complementarity). Moreover, the degree of departure from unity provides immediate quantitative information about the effect on the relative factor share. Thus, the Morishima elasticities using distance function provide immediate quantitative and qualitative comparative-static information about the effect of quantity changes on relative shares.

3. Empirical implementation

3.1 Specification of functional form

Parametric application of the concepts in Section 2 requires specification of a cost function or a distance function. We adopt translog specifications, incorporating technological change (proxied by a time index t) of each[12]:

$$\ln C(p,y,t) = \alpha_0 + \sum_{i=1}^{n} \alpha_i \ln p_i + \sum_{i=1}^{m} \beta_i \ln y_i + \frac{1}{2}\sum_{i=1}^{n}\sum_{j=1}^{n} \alpha_{ij} \ln p_i \ln p_j + \frac{1}{2}\sum_{i=1}^{m}\sum_{j=1}^{m} \beta_{ij} \ln y_i \ln y_j + \sum_{i=1}^{n}\sum_{j=1}^{m} \gamma_{ij} \ln p_i \ln y_j + \theta t + \sum_{i=1}^{n} \nu_i t \ln p_i + \sum_{i=1}^{m} \tau_i t \ln y_i + \varepsilon_{ic} \quad (3.1)$$

and:

$$\ln D(x,y,t) = \alpha_0 + \sum_{i=1}^{n} \alpha_i \ln x_i + \sum_{i=1}^{m} \beta_i \ln y_i + \frac{1}{2}\sum_{i=1}^{n}\sum_{j=1}^{n} \alpha_{ij} \ln x_i \ln x_j + \frac{1}{2}\sum_{i=1}^{m}\sum_{j=1}^{m} \beta_{ij} \ln y_i \ln y_j + \sum_{i=1}^{n}\sum_{j=1}^{m} \gamma_{ij} \ln x_i \ln y_j + \theta t + \sum_{i=1}^{n} \nu_i t \ln x_i + \sum_{i=1}^{m} \tau_i t \ln y_i, + \boldsymbol{e}_{id} \quad (3.2)$$

where the last three terms in each specification reflect technological change and $\boldsymbol{e}_i$ is the stochastic error with zero mean[13]. The corresponding systems of share equations are given by:

$$s_i(p,y,t) = \frac{\partial \ln C(p,y,t)}{\partial \ln p_i} = \alpha_i + \sum_{j=1}^{n} \alpha_{ij} \ln p_j + \sum_{j=1}^{m} \gamma_{ij} \ln y_j + \nu_i t + \boldsymbol{e}_{is}, \quad i = 1, \ldots, n, \quad (3.3)$$

and:

$$\overset{*}{s}_i(x,y,t) = \frac{\partial \ln D(x,y,t)}{\partial \ln x_i} = \alpha_i + \sum_{j=1}^{n} \alpha_{ij} \ln x_j + \sum_{j=1}^{m} \gamma_{ij} \ln y_j + \nu_i t + \boldsymbol{e}_{isstar}, \ i = 1, \ldots, n. \quad (3.4)$$

The homogeneity restrictions on C and D imply the following restrictions in each of these two specifications:

$$\sum_i \alpha_i = 1 \text{ and } \sum_i \alpha_{ij} = \sum_i \gamma_{ij} = \sum_i \nu_i = 0 \quad \forall j. \quad (3.5)$$

One can test for homogeneity by testing for these parametric restrictions. The above specifications of the cost and distance functions impose no restrictions on returns to scale. Constant returns to scale will imposes the following additional restrictions on equation (3.1) or on equation (3.2): $\sum_i \beta_i = 1$ and $\sum_j \beta_{ij} = \sum_j \gamma_{ij} = 0 \quad \forall i$. In order to identify these parameters one needs to estimate a system of share equation with the distance function for the direct framework, which is not trivial, and is not the focus of this paper. Thus, in our case the test for homogeneity is also the test for constant returns to scale. For the translog cost specification the expression for the dual Allen elasticity of substitution is $\sigma_{ij}^{A} = ((\alpha_{ij})/(s_i * s_j)) + 1$ and for Morishima elasticity of substitution it is $\sigma_{ij}^{M} = s_j * \left[\sigma_{ij}^{A} - \sigma_{jj}^{A}\right]$. The direct elasticity of complementarity using the distance function is identical to the dual elasticity using the cost function with the corresponding share equations as $\overset{*}{s}_i(x, y)$.

3.2 Estimation

We apply the foregoing concepts and calculating the nonparametric distribution of the elasticities using annual data on resident and non-resident workers in Switzerland for the 1950-1986 time period. Like rest of the Western countries non-resident labor drastically increased in Switzerland during the 1950s and 1960s as well as there was a steady increase in imports. Kohli (1999) used this data to examine the substitution issues between domestic labor and foreign labor in a production theory approach for Switzerland[14]. This database has only a single (aggregate) output, along with four inputs – resident labor, non-resident labor, imports, and capital. Gross output and import figures are derived from the Swiss National Income and Product Accounts. The quantity of labor is the product of total employment and the average length of the work week. The quantity of capital is calculated as the Törnqvist quantity index of structures and equipment. The income shares of labor and capital are derived from the National Income and Product Accounts, and the prices of labor and capital are obtained by deflation. The resident-worker category comprises natives as well as foreign workers who are residents of Switzerland. Nonresident workers are holders of seasonal permits, annual permits, or transborder permits. We use a time trend with unit annual increments for technological change.

Tables I and II contain estimates of the systems of share equations (3.3) and (3.4), respectively, under alternative assumptions about returns to scale. The subscripts of the coefficients in the first column, L, N, M, K, and Y, represent resident labor, non-resident labor, imports, capital, and output, respectively. Zellner's SURE technique was used to estimate the systems of factor-share equations, and the capital-share equation was deleted for the estimation. Because of possible simultaneous-equations bias, we also estimated three-stage least squares; the coefficients were substantially unchanged in each case. Hausman tests rejects the hypothesis of endogenity of input quantities in the estimation of the share equations in the direct specification and of endogenity of prices in the dual specification. Tests for concavity of the cost function (whence the system (3.3) is derived) were satisfied for 32 of the 37 observations. Concavity of the distance function (whence equation (3.4) is derived) was satisfied at 23 of the 37 observations. Concavity of the cost function is a theoretical imperative. Concavity of the distance function is implied by convexity of input requirement sets $L(y)$, but the distance function is well defined, as are the shadow price functions given by equation (2.10), even if input requirement sets are not convex. On the other hand, the comparative statics of income shares under perfectly competitive pricing of inputs, reflected in equations (2.24) and (2.27), require convexity of input requirement sets as does the estimation of the share equations (2.10) using price data.

We first test for positive linear homogeneity of the production function under each of these specifications. In each case, the critical value of the Wald test statistic for constant returns to scale, under the null of $\gamma_{LY} = \gamma_{NY} = \gamma_{MY} = \gamma_{KY} = 0$, is 9.31. The Wald statistic for the estimates of the systems (3.3) and (3.4) under the complete

Coefficients	Homogeneous technology	Non-homogeneous technology
α_L	0.433 (0.008)	3.469 (0.405)
α_N	0.045 (0.004)	− 1.556 (0.229)
α_M	0.309 (0.004)	0.024 (0.343)
α_K	0.132 (0.002)	0.937 (0.187)
β_{LL}	− 0.424 (0.065)	− 0.051 (0.059)
β_{LN}	0.258 (0.038)	0.089 (0.034)
β_{LM}	0.078 (0.032)	− 0.023 (0.041)
β_{LK}	0.088 (0.113)	− 0.015 (0.110)
β_{NN}	− 0.101 (0.029)	− 0.035 (0.025)
β_{NM}	− 0.101 (0.022)	− 0.012 (0.025)
β_{NK}	− 0.056 (0.07)	− 0.042 (0.125)
β_{MM}	0.081 (0.029)	0.033 (0.045)
β_{MK}	− 0.058 (0.067)	0.002 (0.105)
β_{KK}	− 0.026 (0.069)	0.055 (0.083)
v_L	0.007 (0.002)	0.002 (0.001)
v_N	− 0.008 (0.001)	− 0.005 (0.001)
v_M	0.003 (0.001)	0.004 (0.001)
v_K	− 0.002 (0.005)	− 0.001 (0.001)
γ_{LY}		− 0.248 (0.033)
γ_{NY}		0.131 (0.019)
γ_{MY}		0.023 (0.028)
γ_{KY}		0.094 (0.065)

Note: Standard errors in parentheses

Table I. Estimated coefficients of the translog cost function

restrictions imposed by are 100.2 and 29.1, respectively; thus, positive linear homogeneity is decisively rejected in each case.

Tables III and IV contain estimates of the dual Allen-Uzawa and Morishima elasticities of substitution (equations (2.11) and (2.14)), and Tables V and VI contain estimates of the direct elasticities (equations (2.22) and (2.20)), all evaluated at the means of the variables and under alternative assumptions about returns to scale.

Let us first evaluate the qualitative information in these tables. Recall that two factors are classified as dual (Allen-Uzawa or Morishima) substitutes if the dual elasticity is positive and as complements if it is negative, and the reverse is true for the direct elasticities. Of course, the non-symmetry of the Morishima elasticities raises the possibility of ambiguities in the Morishima taxonomy. Examination of the tables, however, reveals that there are only two qualitative asymmetries, for capital and imports in the direct homogeneous case (Table V) and for capital and nonresident labor in the direct non-homogeneous framework (Table VI), and in each case one of the two elasticity estimates is statistically insignificant. Hence, it is possible to construct an unambiguous taxonomy of (dual and direct) substitutability and complementarity for the Morishima elasticities as well as the Allen elasticities (evaluated at the means of the data). This taxonomy is summarized in Table VII, where *S* signifies (dual or direct) substitutability and *C* signifies (dual or direct) complementarity.

The strongest priors exist for resident and non-resident labor, and, indeed, the two are substitutes under either assumption about returns to scale, in either the primal or dual specification, and with respect to either elasticity definition[15]. Thus, an increase in the number of non-resident workers lowers the wage rate of resident workers,

Coefficients	Homogeneous technology	Non-homogeneous technology
α_L	0.291 (0.007)	0.431 (0.034)
α_N	0.156 (0.005)	0.146 (0.017)
α_M	0.27 (0.009)	0.275 (0.033)
α_K	0.283 (0.01)	0.148 (0.02)
β_{LL}	0.070 (0.009)	0.084 (0.011)
β_{LN}	− 0.068 (0.005)	− 0.068 (0.006)
β_{LM}	− 0.021 (0.01)	0.002 (0.01)
β_{LK}	− 0.02 (0.01)	− 0.018 (0.04)
β_{NN}	0.039 (0.003)	0.039 (0.004)
β_{NM}	− 0.004 (0.006)	0.01 (0.006)
β_{NK}	− 0.033 (0.01)	0.019 (0.01)
β_{MM}	0.11 (0.017)	0.105 (0.016)
β_{MK}	− 0.085 (0.03)	− 0.118 (0.02)
β_{KK}	− 0.02 (0.02)	0.154 (0.02)
v_L	− 0.0002 (0.001)	− 0.001 (0.001)
v_N	− 0.001 (0.0004)	− 0.002 (0.0005)
v_M	− 0.001 (0.001)	0.001 (0.0009)
v_K	0.002 (0.001)	0.002 (0.002)
γ_{LY}		− 0.102 (0.23)
γ_{NY}		0.004 (0.011)
γ_{MY}		0.001 (0.023)
γ_{KY}		− 0.097 (0.232)

Note: Standard errors in parentheses

Table II.
Estimated coefficients of the translog distance function

	Allen-Uzawa				Morishima			
	L	*N*	*M*	*K*	*L*	*N*	*M*	*K*
L	− 3.72 (0.619)	10.44 (3.903)	1.66 (0.067)	1.89 (0.085)		3.15 (2.009)	0.89 (0.082)	1.32 (0.221)
N		− 38.41 (44.82)	− 4.67 (3.317)	− 2.73 (2.089	6.00 (0.771)		− 0.87 (0.813)	0.24 (0.811)
M			− 1.54 (0.132)	0.11 (0.053)	2.28 (0.035)	2.18 (0.822)		0.90 (0.18)
K				− 3.78 (0.256)	2.38 (0.022)	2.30 (0.455)	0.46 (0.014)	

Note: Standard errors in parentheses

Table III.
Dual elasticities of substitution using the cost function: homogeneous technology

both absolutely and relatively to the wage rate of non-resident workers. And an increase in the wage rate of resident workers increases the demand for non-resident workers, both absolutely and relatively to the demand for resident workers.

Some differences emerge with other pairs of inputs. First, regarding the assumption about returns to scale, there are a couple of reversals using the dual Allen elasticity definition and one reversal for the direct under both elasticity definitions. Since the constant-returns-to-scale cost and distance functions are misspecified, we would conclude that resident labor and imports are dual Allen-Uzawa substitutes and that resident labor and capital are dual Allen-Uzawa substitutes. On the other hand non-resident labor and imports are dual Allen-Uzawa substitutes and are direct complements under both definition of elasticity. Hence, an increase in the number of non-resident labor increases the price of imports, both absolutely and relatively. Whereas, an increase in the wage rate of non-resident workers increase the demand for imports. Similarly, we would conclude that non-resident labor and capital are direct substitutes under either elasticity definition. Hence, an increase in the number of non-resident workers lowers the rental rate on capital, both absolutely and relatively to the wage rate of non-resident workers.

Table IV. Dual elasticities of substitution using the cost function: non-homogeneous technology

	Allen-Uzawa				Morishima			
	L	*N*	*M*	*K*	*L*	*N*	*M*	*K*
	−1.65	4.26	0.81	0.85		1.74	0.83	0.73
L	(0.031)	(0.404)	(0.027)	(0.007)		(0.195)	(0.002)	(0.017)
		−22.69	0.33	−1.8	2.50		0.70	0.11
N		(0.004)	(0.002)	(0.064)	(0.002)		(0.001)	(0.002)
			−2.16	−0.81	1.04	1.48		0.34
M			(0.014)	(0.009)	(0.016)	(0.007)		(0.008)
				−2.28	1.06	1.35	0.38	
K				(0.122)	(0.007)	(0.040)	(0.037)	

Note: Standard errors in parentheses

Table V. Direct elasticities of substitution using the distance function: homogeneous technology

	Allen-Uzawa				Morishima			
	L	*N*	*M*	*K*	*L*	*N*	*M*	*K*
	−0.41	−1.49	0.82	0.80		−0.12	0.32	0.23
L	(0.02)	(0.26)	(0.03)	(0.01)		(0.126)	(0.256)	(0.016)
		0.33	0.78	0.80	−0.46		0.31	0.23
N		(0.99)	(0.01)	(0.06)	(0.028)		(0.025)	(0.024)
			−0.33	−0.32	0.52	0.15		−0.04
M			(0.01)	(0.01)	(0.015)	(0.016)		(0.001)
				−0.17	0.51	0.03	0.003	
K				(0.03)	(0.034)	(0.015)	(0.007)	

Note: Standard errors in parentheses

Table VI. Direct elasticities of substitution using the distance function: non-homogeneous technology

	Allen-Uzawa				Morishima			
	L	*N*	*M*	*K*	*L*	*N*	*M*	*K*
	−0.38	−1.50	1.02	0.82		−0.08	0.51	0.21
L	(0.031)	(0.027)	(0.027)	(0.007)		(0.195)	(0.002)	(0.017)
		−0.32	1.55	−0.28	−0.47		0.15	−0.04
N		(0.004)	(0.002)	(0.064)	(0.002)		(0.001)	(0.002)
			−0.81	−0.82	0.59	0.12		−0.17
M			(0.014)	(0.009)	(0.008)	(0.007)		(0.001)
				−0.10	0.51	0.003	−0.003	
K				(0.02)	(0.036)	(0.037)	(0.037)	

Note: Standard errors in parentheses

Table VII. Taxonomy for substitutes and complements

	Dual				Direct			
	Homogeneous technology		Non-homogeneous technology		Homogeneous technology		Non-homogeneous technology	
	AES	MES	AES	MES	AES	MES	AES	MES
LN	S	S	S	S	S	S	S	S
LM	S	S	S	S	S	C	C	C
LK	S	S	S	S	C	C	C	C
NM	C	S	S	S	C	C	C	C
NK	C	S	C	S	C	C	S	S
MK	S	S	C	S	S	S	S	S

It should be re-emphasized that contrasts in the classification of pairs as substitutes or complements in the primal and the dual do not constitute any kind of theoretical or econometric inconsistency: the direct and dual elasticities measure conceptually distinct phenomena when there are more than two inputs[16]. The dual elasticities measure the effect on quantities of price changes whereas the direct elasticities measure the effect on prices of changes in quantities.

Finally, regarding the difference in taxonomies induced by the two elasticity concepts, it is clear that the dual Morishima elasticity concept is more conducive to substitutability than the dual Allen-Uzawa elasticity, as shown theoretically in Section 2. In some (dual elasticity) cases, pairs are classified as complements by the Allen elasticity concept but as substitutes by the Morishima concept. Interestingly, however, there is no difference in the classification scheme in the dual. Again, we wish to re-emphasize that there is no theoretical or econometric reason why the two elasticity concepts should yield comparable qualitative conclusion about substitutability/complementarity; they are measuring different concepts, as discussed in Section 2.

3.3 Elasticities reported at the mean

To consider the quantitative elasticity results, we take note of the fact that the Allen-Uzawa elasticity of substitution has no meaningful quantitative interpretation, as pointed out by Blackorby and Russell (1975, 1989). The size of the simple cross price elasticity $\varepsilon_{ij}(p, y)$ is meaningful, but dividing by the share of input j, as in equation (2.11), to obtain the Allen-Uzawa elasticity, $\sigma_{ij}^{A}(p, y)$, undermines this quantitative content. Thus, to facilitate consideration of quantitative comparative statics, we list the price elasticities and their duals, defined by equations (2.12) and (2.21) and evaluated at the means of the data, in Table VIII. Of course, these elasticities are non-symmetric.

Not surprisingly, from Table VIII we see that the quantitative estimate of elasticity at the mean is different under the two specification of the technology. We focus on the quantitative relationships between the two types of labor under the (preferred) non-homogeneous specification of the technology. The estimated cross price elasticities in Table VIII indicate that a 1-percent increase in the wage rate of non-resident workers would increase the employment of resident labor by 0.3 percent (at the means of the data), whereas a 1-percent increase in the wage rate of resident labor would increase the employment of non-resident labor by 1.8 percent[17]. Table III indicates that a 1-percent increase in the price of non-resident labor would increase the quantity ratio of resident

	L	N	M	K
Dual price elasticity: homogeneous technology				
L	−1.577	0.674	0.463	0.44
N	4.423	−2.5	−1.286	−0.636
M	0.703	−0.297	−0.431	0.025
K	0.802	−0.176	0.030	−0.879
Dual price elasticity: non-homogeneous technology				
L	−0.697	0.275	0.225	0.197
N	1.803	−1.477	0.093	−0.418
M	0.341	0.022	−0.603	0.240
K	0.359	−0.116	0.288	−0.531
Direct quantity elasticity: homogeneous technology				
L	−0.411	−0.096	0.230	0.185
N	−0.631	−0.33	0.217	−0.279
M	0.348	0.050	−0.327	−0.072
K	0.338	−0.077	−0.086	−0.853
Direct quantity elasticity: non-homogeneous technology				
L	−0.614	−0.096	0.284	0.19
N	−0.631	−0.33	0.434	0.527
M	0.431	0.1	−0.345	−0.190
K	0.346	0.146	−0.228	−0.105

Table VIII. Price and quantity elasticities ε_{ij} and ε_{ij}^{*}

labor to non-resident labor by 1.7 percent, while a 1-percent increase in the wage rate of resident labor would increase the quantity ratio of non-resident to resident labor by 2.5 percent[18].

The direct price elasticity estimates in Table VIII suggest that a 1-percent increase in the number of nonresident workers would lower the wage rate of resident workers by 0.1 percent, while a 1-percent increase in the number of resident workers would lower the wage rate of non-resident workers by 0.6 percent. From Table VI, it appears that a 1-percent increase in the number of non-resident workers would lower the relative wage rate (of non-resident to resident workers) by just 0.1 percent, while a 1-percent increase in the number of resident workers would lower the relative wage rate (of resident to non-resident workers) by 0.5 percent.

The estimated dual Allen-Uzawa elasticity of substitution does provide immediate qualitative information about absolute shares, as reflected in equation (2.16). Thus, the elasticity of 4.3 in Table IV indicates that the share of either type of labor input is enhanced by an increase in the wage rate of the other labor type. Similarly, from (2.24) and Table VI, we see from the negative sign of the direct Allen-Uzawa elasticity that an increase in the quantity of either input decreases the absolute share of the other labor input. Equations (2.22) and (2.27), along with the estimates of Morishima elasticities in Tables IV and VI, provide both qualitative and quantitative information about the comparative statics of relative income shares. Thus, from Table IV, we see that a 1-percent increase in the wage rate of non-resident workers increases the share of resident workers relative to non-resident workers by 0.7 percent, while a 1-percent increase in the wage rate of resident workers increases the share of non-resident workers relative to resident workers by 1.5 percent[19]. The direct Morishima elasticities provide information about the effect on relative shares of changes in input quantities. Table VI indicates that a 1-percent increase in the quantity of non-resident labor decreases the share of resident labor relative to non-resident labor by 1.1 percent, whereas a 1-percent increase in the quantity of resident labor decreases the share of non-resident labor relative to resident labor by 1.5 percent.

3.4 Nonparametric distribution of the elasticities

All of the foregoing calculations and interpretations are carried out at the means of the data. While this is a standard way of reporting elasticity results, it should be emphasized that the interpretations could be egregiously in error for any particular year, unless the elasticities were constant over time and over the domain of the cost or distance function. Even where the estimated standard errors, also calculated at the mean, are small relative to the elasticity size, there is no reason to believe that the elasticity is time invariant or insensitive to the values of the input quantities or prices. For a particular question about a particular year, one can calculate the appropriate elasticity. For the purpose of this paper, we summarize the information about the elasticities in two ways. First, Table IX lists the ranges of the elasticity estimates (under the non-homothetic technology specifications). It can be seen that the range is fairly tight for some pairs of inputs for both elasticity concepts: namely, resident labor and imports, resident labor and capital, and imports and capital. On the other hand, the range is quite large for other pairs. Note, in particular, that the (qualitative) classification of two inputs as substitutes or complements is itself sensitive to the choice of the sample point at which to do the calculation in the cases of the dual Allen elasticity for non-resident labor and imports, the dual Morishima elasticity for non-resident labor and capital, and the direct Morishima elasticity for resident and non-resident labor and for non-resident labor and imports.

A second way to summarize the information about the elasticity values over the entire sample space is frequency distributions. Frequency distributions are useful because they give the probability of the elasticity for various elasticity intervals, which might be useful for policy purposes over and above summary information such as a range or an index. We have constructed nonparametric, kernel-based density estimates of the distributions of each of the elasticities (essentially "smoothed" histograms of elasticity values) over the entire sample (numbering 72 distributions)[20]. While many of the distributions look radically different for the linear homogeneous and non-homogeneous cases,

	Dual				Direct			
	Non-homogeneous technology				Non-homogeneous technology			
	AES		MES		AES		MES	
	Min.	Max.	Min.	Max.	Min.	Max.	Min.	Max.
LN	3.15	8.15	1.56	2.56	−4.46	−0.64	−0.68	0.47
NL			1.94	4.63			−1.86	0.16
LM	0.78	0.84	0.81	0.85	1.01	1.02	0.55	0.69
ML			1.00	1.06			0.78	0.84
LK	0.83	0.88	0.81	0.85	0.79	0.85	0.19	0.35
KL			1.04	1.08			0.04	0.21
NM	−0.96	0.61	0.39	0.77	1.32	2.63	−0.5	0.68
MN			1.27	2.34			−0.5	0.68
NK	−6.32	−0.59	−0.91	0.39	1.72	4.31	0.52	1.11
KN			1.14	2.21			−0.46	0.73
MK	1.03	1.04	0.73	0.79	−0.64	−0.8	−0.16	−0.03
KM			0.86	0.91			0.05	0.16

Table IX.
Range for the dual and direct elasticity

some look similar. This raises the possibility that, even though the hypothesis of constant returns to scale is easy to reject (at least for this data set), the apparent misspecification might have little effect on estimated elasticity values and hence on qualitative and quantitative comparative statics of price, quantities, and income shares. We test this hypothesis formally by testing for the difference between the elasticity distributions under constant returs to scale vs non constant returns to scale technology. In particular, Fan and Ullah (1999) have proposed a nonparametric (time series) test for the comparison of two unknown distributions, say f and g – that is, a test of the null hypothesis, $H_0 : f(x) = g(x)$ for all x, against the alternative, $H_1 : f(x) \neq g(x)$ for some x[21]. Tables AI and AII contain the results of carrying out these tests. The hypothesis that the two distributions are identical is rejected in every case but one: the dual Allen elasticity between non-resident labor and capital. Thus, it is safe to say that the misspecification of constant returns to scale, required for the Hicks (1970) and Sato and Koizumi (1973) dual elasticity concept results in serious errors in elasticity estimates and hence in serious errors in the comparative statics of prices, quantities, and income shares.

4. Summary and concluding remarks

Using a Swiss dataset this paper empirically estimates the elasticity of substitution in the dual framework to asses the effect of price change on the input quantity using the translog cost function and in the primal framework to asses the effect of quantity change on price using the translog distance function translog distance function. We reconcile and summarize the Allen-Uzawa and Morishima elasticity in the primal and dual framework. We also test and reject the hypothesis of homotheticity under two different specifications of the technology. Maintaining a non-homogeneous translog (cost or distance function) technology, we find that misspecification of the technology as homogeneous of degree one results in statistically significant errors in the estimated elasticities of substitution and hence in assessments of the effects on input demands, prices, or shares of changes in quantities or prices.

This paper demonstrates that reporting elasticity of substitution at the mean, the usual practice in the literature, might give incorrect conclusion regarding whether the inputs are substitutes or complements as well as the degree of substitution. We not only give the range for the elasticity estimates but also plot nonparametric distributions for the elasticity estimated under homogenous and non-homogenous technology. We find that the dual Allen elasticity for non-resident labor and imports, the dual Morishima elasticity for non-resident labor and capital, and the direct Morishima elasticity for resident and non-resident labor and for non-resident labor and imports are sensitive to the data point where we are calculating these elasticities. In all the cases except for the dual Allen elasticity between non-resident labor and capital the equality of two distributions under homogenous vs non-homogenous technology is rejected.

Overall we find some interesting elasticity estimates for non-resident labor for Switzerland during this time period. Using a production theory open economy model we find that the foreign and domestic labor are substitutes for Switzerland. But the magnitude of the elasticity for these two kind of labor in the dual and direct framework is very different. A one percent increase in the quantity of non-resident worker lowers the wages of resident workers by 0.1 percent. Whereas, from Morishima dual elasticity we find that a one percent increase in the wages of nonresident workers increase the share of resident workers relative to non-resident workers by 0.7 percent.

Non-resident labor and imports are complements – so more foreign workers will increase the price for imports and an increase in the wage rate of non-resident workers will increase the demand for imports. Whereas, non-resident labor and capital are direct substitutes, though switch between Allen complement and Morishima substitute in the dual framework. These findings shows that when examining the effect of foreign labor in production one needs to estimate various measures of elasticity.

Notes

1. Davis and Shumway (1996) show that the Morishima elasticities are the correct measure of the percentage change in relative factors for a percentage change in price, isoquant curvature, and changes in relative factor shares for changes in relative prices only when the technology is homethetic.
2. Even if this representation of technology is well known it is important to mention it here to help the reader, particularly who are new to this literature.
3. These assumptions are stronger than needed for much of the conceptual development that follows, but in the interest of simplicity we maintain them throughout.
4. $L(y) = L(y) + \mathbf{R}^n_+ \forall y \in \mathbf{R}^m_+$.
5. $\geq y \Rightarrow L(\,) \subset L(y)$.
6. $y \neq 0^{(m)} \Rightarrow 0^{(n)} \notin L(y)$.
7. We restrict the domain of the distance function to assure that it is globally well defined. An alternative approach (Färe and Primont, 1995) is to define D on the entire non-negative $(n + m)$-dimensional Euclidean space and replace "max" with "sup" in the definition. See Russell (1997) for a comparison of these approaches.
8. Whatever is not there can be found in Diewert (1974) or the Fuss and McFadden (1978) volume.
9. See Färe and Grosskopf (1994) and Russell (1997) for analyses of the distance function and associated shadow prices.
10. The dual Allen-Uzawa and Morishima elasticities of substitution are idential when $n = 2$, as are the direct Allen-Uzawa and Morishima elasticities.
11. While shadow prices and direct elasticities are well defined even if the input requirement sets are not convex, the comparative statics of income shares using these elasticities requires convexity (as well, of course, as price-taking, cost-minimizing behavior), which implies concavity of the distance function in x. By way of contrast, convexity of input requirement sets is not required for the comparative statics of income shares using direct elasticities, since the cost function is necessarily concave in prices. See Russell (1997) for a discussion of these issues.
12. We choose these two specifications because of their flexibility (in the sense of both Diewert (1971) and Jorgenson and Lau (1975)). The two specifications represent different technologies; that is, the translog is not self-dual. Moreover, it is not possible to find closed-form duals to either of these specifications, unless they degenerate to representations of a Cobb-Douglas technology (which is self-dual). Of course, the stochastic structure is also quite different in the two specifications.
13. We choose the same notation for the parameters in the two translog specification for the ease of reporting the estimated results.
14. We thank Ulrich Kohli for providing these data.
15. See the finding of substitutability between immigrant and native workers for the USA by Grossman (1982).

16. As noted earlier, they require different theoretical and stochastic specifications of the technology as well.
17. It is interesting to note here that, under the (apparently misspecified) homogeneous technology, a 1-percent increase in the wage rate of resident labor would increase the employment of non-resident labor by a whopping 4.4 percent, an estimate that strains credibility.
18. Again, note that under the misspecified homogeneous technology, a 1-percent change in a wage rate generates an estimated 3-percent or 6-percent change in the quantity ratio, again challenging our intuition.
19. Note that, under the (misspecified) homogeneous technology, a 1-percent increase in the wage rate of resident workers increases the share of non-resident workers by a hard-to-believe 5 percent.
20. See the Appendix for the particulars. We report 36 distributions in the paper and others are available from the author upon request.
21. See the Appendix for an exact description of the test statistic.

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Appendix

Each of the distributions in Figures A1-A6 is a kernel-based estimate of a density function, $f(\cdot)$, of a random variable x, based on the standard normal kernel function and optimal bandwidth:

$$\hat{f}(x) = \frac{1}{nh}\sum_{j=1}^{J} k\left(\frac{x_j - x}{h}\right),$$

where $\int_{-\infty}^{\infty} k(\psi)d\psi = 1$ and $\psi = (x_j - x)/h$. In this construction, h is the optimal window width, which is a function of the sample size n and goes to zero as $n \rightarrow \infty$. We assume that k is a symmetric standard normal density function, with non-negative images. The optimal window width is chosen by minimizing the mean integrated square error. See Pagan and Ullah (1999) for details.

The statistic used to test for the difference between two distributions, predicated on the integrated-square-error metric on a space of density functions, $I(f,g) = \int_x (f(x) - g(x))^2 dx$, is:

$$T = \frac{nh^{1/2}\hat{I}}{\hat{\sigma}} \sim N(0,1), \tag{A1}$$

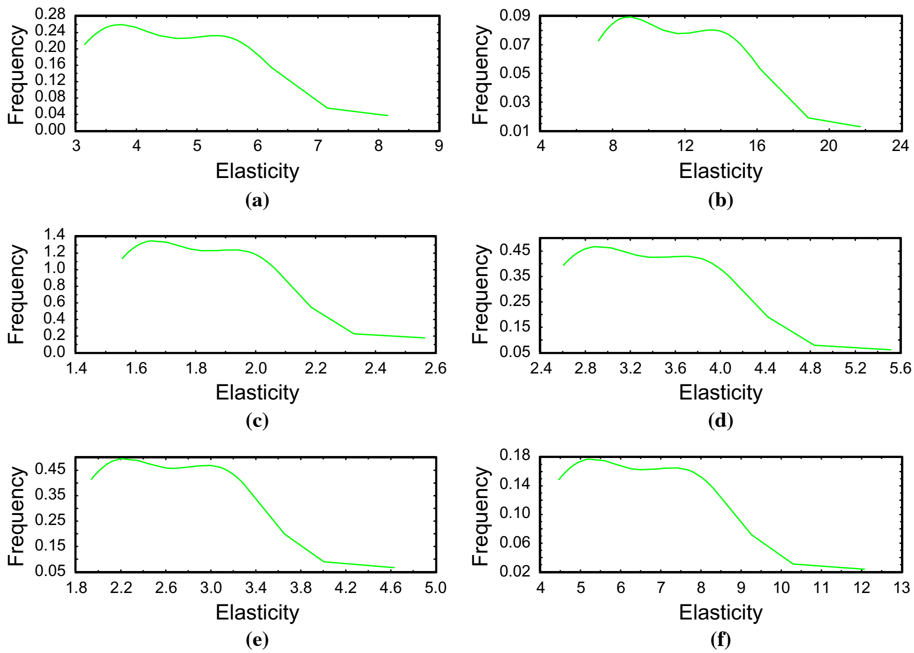


Figure A1.
Distributions of dual Allen and Morishima elasticities: non-resident labor and resident labor

Notes: (a) Allen-Uzawa elasticity of substitution between non-resident labor and resident labor with non-homogeneous technology; (b) Allen-Uzawa elasticity of substitution between non-resident labor and resident labor with homogeneous technology; (c) Morishima elasticity of substitution between resident labor and non-resident labor with non-homogeneous technology; (d) Morishima elasticity of substitution between resident labor and non-resident labor with homogeneous technology; (e) Morishima elasticity of substitution between non-resident labor and resident labor with non-homogeneous technology; (f) Morishima elasticity of substitution between non-resident labor and resident labor with homogeneous technology

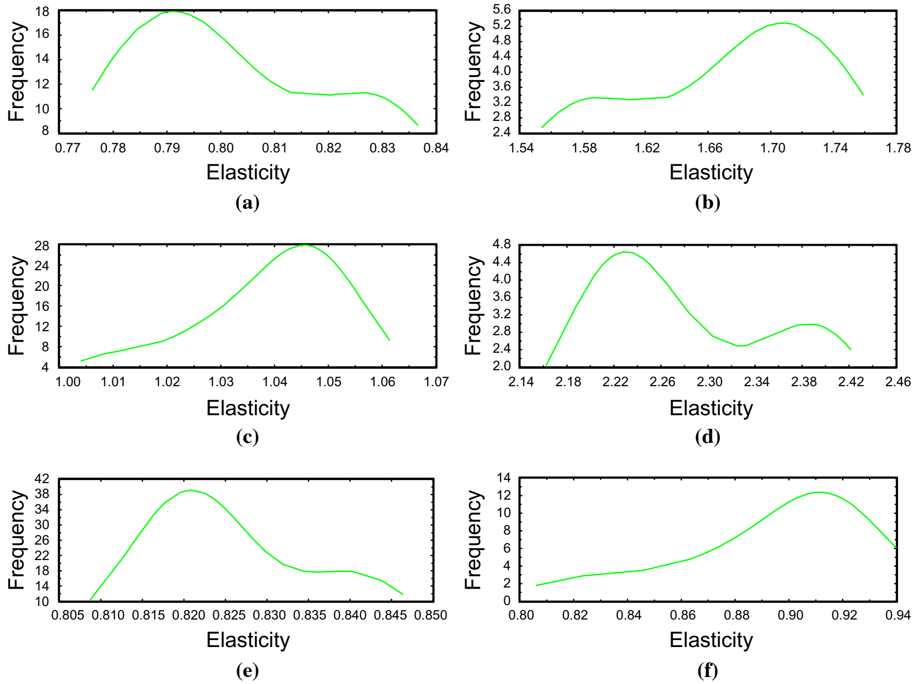
where:

$$\hat{I} = \frac{1}{n^2 h} \sum_{\substack{i=1 \\ (i \neq j)}}^{n} \sum_{j=1}^{n} \left[k\left(\frac{x_i - x_j}{h}\right) + k\left(\frac{y_i - y_j}{h}\right) - 2k\left(\frac{y_i - x_j}{h}\right) - k\left(\frac{x_i - y_j}{h}\right) \right] \quad \text{(A2)}$$

and:

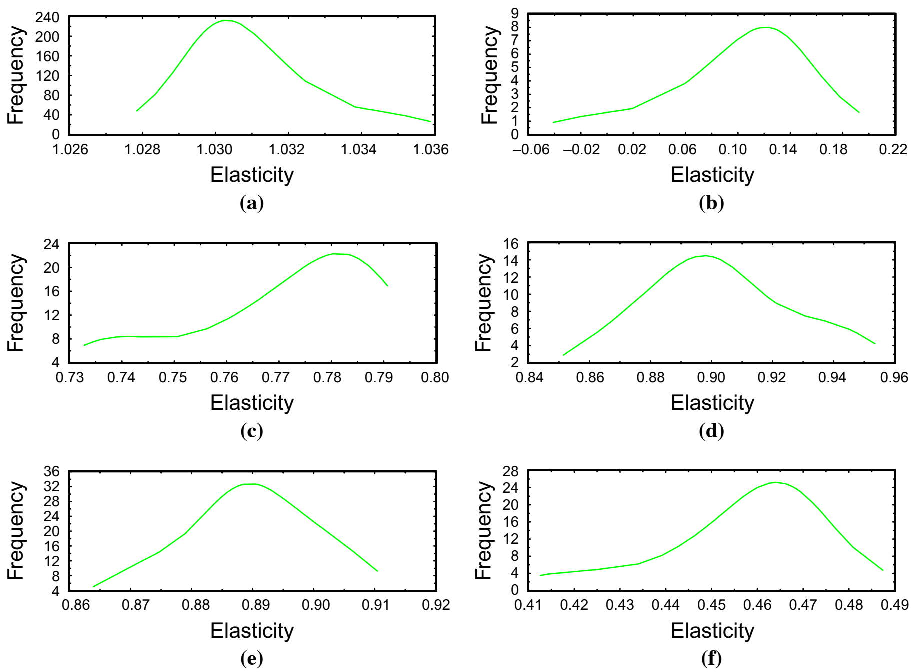
$$\hat{\sigma} = \frac{2}{n^2 h} \sum_{i=1}^{n} \sum_{j=1}^{n} \left[k\left(\frac{x_i - x_j}{h}\right) + k\left(\frac{y_i - y_j}{h}\right) + 2k\left(\frac{x_i - y_j}{h}\right) \right] \int k^2(\Psi) d\psi. \quad \text{(A3)}$$

As shown by Fan and Ullah (1999), the test statistic asymptotically goes to the standard normal, but the sample in our study is only 37 years. Thus, we do a bootstrap approximation with 2,000 replications to find the critical values for the statistic at the 5-percent and 1-percent levels of significance (Tables AI-AIV).



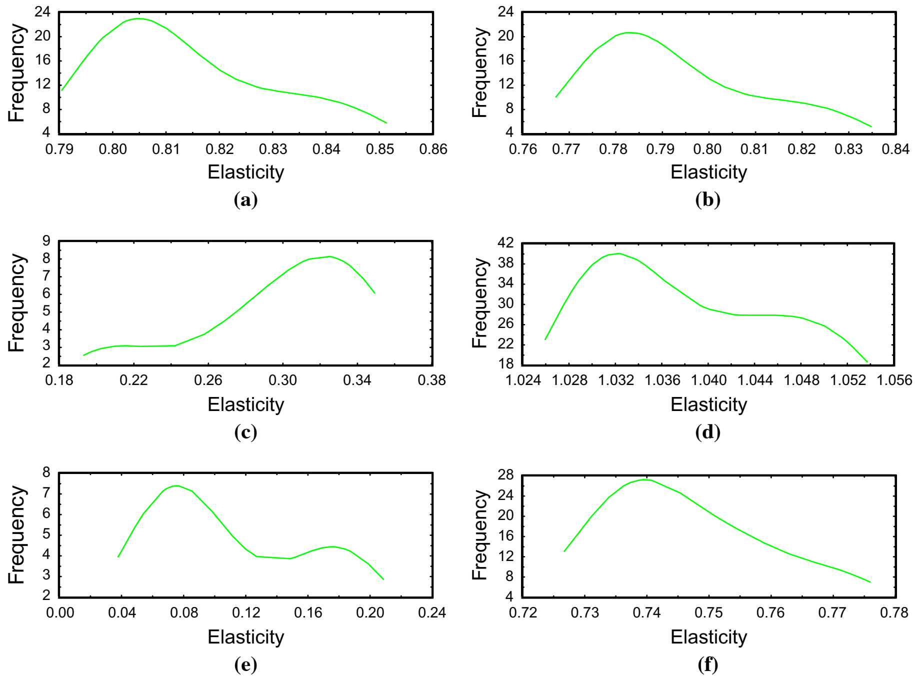
Notes: (a) Allen-Uzawa elasticity of substitution between imports and resident labor with non-homogeneous technology; (b) Allen-Uzawa elasticity of substitution between imports and resident labor with homogeneous technology; (c) Morishima elasticity of substitution between imports and resident labor with non-homogeneous technology; (d) Morishima elasticity of substitution between imports and resident labor with homogeneous technology; (e) Morishima elasticity of substitution between resident labor and imports with non-homogeneous technology; (f) Morishima elasticity of substitution between resident labor and imports with homogeneous technology

Figure A2.
Distributions of dual Allen and Morishima elasticities: imports and resident labor



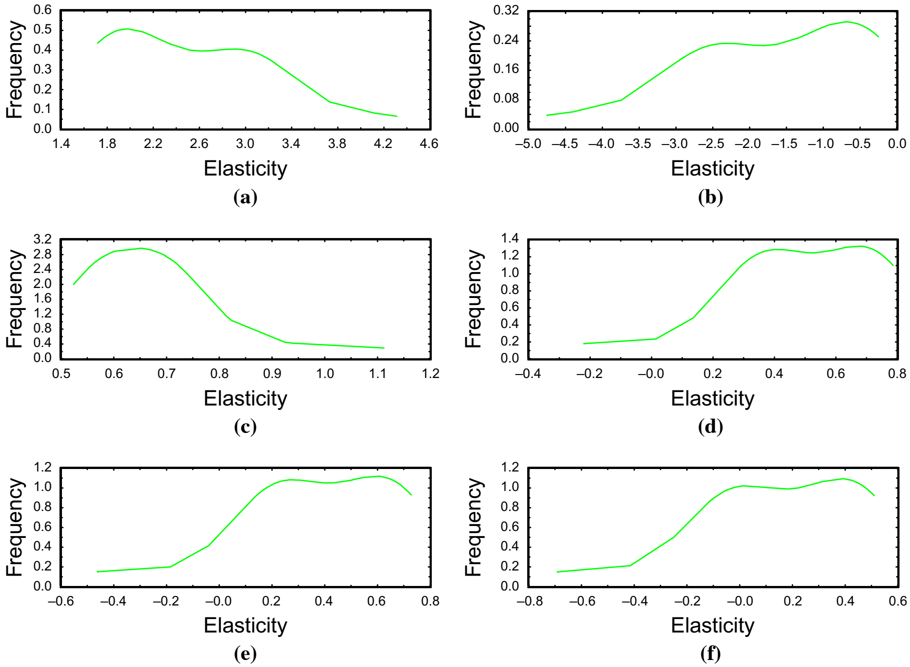
Notes: (a) Allen-Uzawa elasticity of substitution between imports and capital with non-homogeneous technology; (b) Allen-Uzawa elasticity of substitution between imports and capital with homogeneous technology; (c) Morishima elasticity of substitution between imports and capital with non-homogeneous technology; (d) Morishima elasticity of substitution between imports and capital with homogeneous technology; (e) Morishima elasticity of substitution between capital and imports with non-homogeneous technology; (f) Morishima elasticity of substitution between capital and imports with homogeneous technology

Figure A3.
Distributions of dual Allen and Morishima elasticities: imports and capital



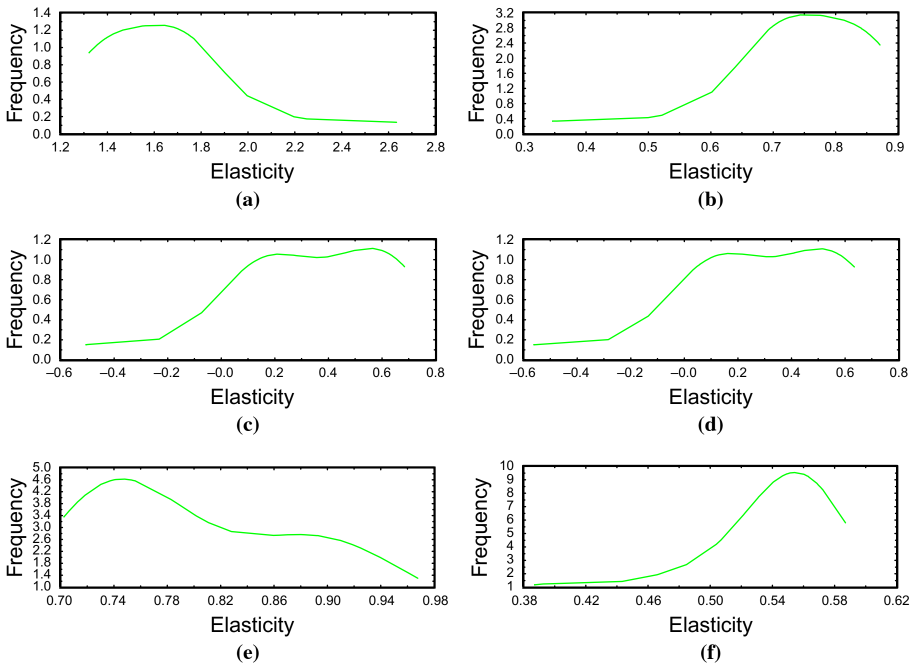
Notes: (a) Allen-Uzawa elasticity of substitution between resident labor and capital with non -homogeneous technology; (b) Allen-Uzawa elasticity of substitution between resident labor and capital with homogeneous technology; (c) Morishima elasticity of substitution between resident labor and capital with non-homogeneous technology; (d) Morishima elasticity of substitution between resident labor and capital with homogeneous technology; (e) Morishima elasticity of substitution between capital and resident labor with non-homogeneous technology; (f) Morishima elasticity of substitution between capital and resident labor with homogeneous technology

Figure A4.
Distributions of direct Allen and Morishima elasticities: resident labor and capital



Notes: (a) Allen-Uzawa elasticity of substitution between non-resident labor and imports with non-homogeneous technology; (b) Allen-Uzawa elasticity of substitution between non-resident labor and capital with homogeneous technology; (c) Morishima elasticity of substitution between non-resident labor and capital with non-homogeneous technology; (d) Morishima elasticity of substitution between non-resident labor and capital with homogeneous technology; (e) Morishima elasticity of substitution between capital and non-resident labor with non-homogeneous technology; (f) Morishima elasticity of substitution between capital and non-resident labor with homogeneous technology

Figure A5.
Distributions of direct Allen and Morishima elasticities: non-resident labor and capital



Notes: (a) Allen-Uzawa elasticity of substitution between non-resident labor and imports with non-homogeneous technology; (b) Allen-Uzawa elasticity of substitution between non-resident labor and imports with homogeneous technology; (c) Morishima elasticity of substitution between non-resident labor and imports with non-homogeneous technology; (d) Morishima elasticity of substitution between non-resident labor and imports with homogeneous technology; (e) Morishima elasticity of substitution between non-resident labor and imports with non-homogeneous technology; (f) Morishima elasticity of substitution between non-resident labor and imports with homogeneous technology

Figure A6.
Distributions of direct Allen and Morishima elasticities: non-resident labor and import

	Test statistic	5 percent significance level Lower critical value	5 percent significance level Upper critical value	1 percent significance level Lower critical value	1 percent significance level Upper critical value	
LN	0.402	0.1534	0.2292	0.1402	0.2439	Reject
ML	27.56	0.0024	0.0035	0.0023	0.0036	Reject
MK	330.49	0.0002	0.0004	0.0002	0.0004	Reject
NK	0.081	0.0426	0.0913	0.0396	0.1062	Fail to reject
NM	1.67	0.0411	0.0918	0.0348	0.0988	Reject

Table AI.
Distribution hypothesis tests: dual Allen elasticity of substitution

Table AII.
Distribution hypothesis tests: dual Morishima elasticity of substitution

	Test statistic	5 percent level of significance Lower critical value	Upper critical value	1 percent level of significance Lower critical value	Upper critical value	
LN	2.19	0.0295	0.0511	0.0269	0.0546	Reject
NL	0.802	0.0792	0.1300	0.0723	0.1381	Reject
MK	30.78	0.0023	0.0034	0.0021	0.0036	Reject
KM	51.74	0.0013	0.0021	0.0012	0.0036	Reject
ML	39.83	0.0017	0.0028	0.0015	0.0031	Reject
LM	50.97	0.0011	0.0018	0.001	0.0018	Reject
NK	0.324	0.0066	0.0167	0.0057	0.0182	Reject
KN	1.762	0.0271	0.0461	0.0251	0.0495	Reject
MN	1.24	0.0207	0.0340	0.0192	0.0364	Reject
NM	6.06	0.0107	0.0206	0.0097	0.0221	Reject

Table AIII.
Distribution hypothesis tests: direct Allen elasticity of substitution

	Test statistic	5 percent level of significance Lower critical value	Upper critical value	1 percent level of significance Lower critical value	Upper critical value	
NM	5.00	0.0134	0.0311	0.0117	0.0332	Reject
NL	0.4	0.1498	0.2287	0.1360	0.2439	Reject
LK	14.38	0.0007	0.0019	0.0007	0.0023	Reject
NK	0.81	0.0078	0.0197	0.0068	0.0236	Reject
MK	7.78	0.0089	0.0161	0.0076	0.0175	Reject
ML	316.98	0.0002	0.0003	0.0002	0.0003	Reject

Table AIV.
Distribution hypothesis tests: direct Morishima elasticity of substitution

	Test statistic	5 percent level of significance Lower critical value	Upper critical value	1 percent level of significance Lower critical value	Upper critical value	
NM	12.42	0.0057	0.0115	0.0042	0.0122	Reject
MN	0.03	0.0006	0.0016	0.0005	0.0019	Reject
LN	2.19	0.0286	0.0509	0.0261	0.0541	Reject
NL	0.8	0.0766	0.1296	0.0697	0.1378	Reject
LK	65.49	0.0011	0.0016	0.001	0.0017	Reject
KL	41.2	0.0017	0.0025	0.0015	0.0027	Reject
NK	1.94	0.007	0.0159	0.0061	0.0177	Reject
KN	0.43	0.0078	0.0197	0.0068	0.0236	Reject
MK	16.03	0.0038	0.0051	0.0035	0.0054	Reject
KM	23.69	0.0026	0.0044	0.0023	0.0047	Reject
ML	37.18	0.0017	0.0021	0.0016	0.0022	Reject
LM	9.8	0.0027	0.0038	0.0026	0.004	Reject
NK	1.94	0.007	0.0159	0.0061	0.0177	Reject
KN	0.43	0.0078	0.0197	0.0068	0.0236	Reject

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